

PROJECT #2 SPACE VECTOR PWM INVERTER

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1. Problem Description

In this simulation, we will study Space Vector Pulse Width Modulation (SVPWM) technique. We will use the SEMIKRON® IGBT Flexible Power Converter for this purpose. The system configuration is given below:

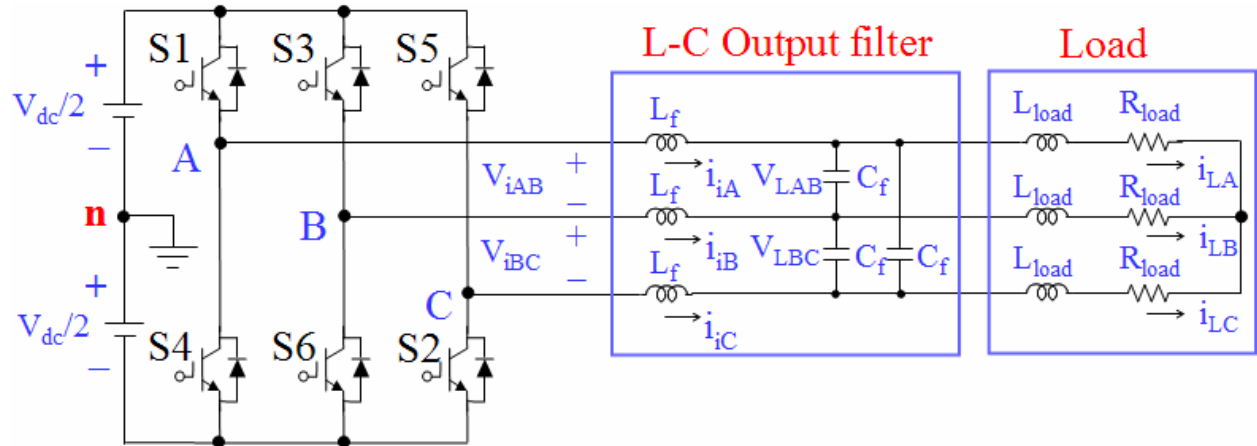


Fig. 1 Circuit model of three-phase PWM inverter with a center-taped grounded DC bus.

The system parameters for this converter are as follows:

- IGBTs: SEMIKRON SKM 50 GB 123D, Max ratings: $V_{CES} = 600 \text{ V}$, $I_C = 80 \text{ A}$
- DC- link voltage: $V_{dc} = 400 \text{ V}$
- Fundamental frequency: $f = 60 \text{ Hz}$
- PWM (carrier) frequency: $f_z = 3 \text{ kHz}$
- Modulation index: $a = 0.6$
- Output filter: $L_f = 800 \mu\text{H}$ and $C_f = 400 \mu\text{F}$
- Load: $L_{load} = 2 \text{ mH}$ and $R_{load} = 5 \Omega$

Using Matlab/Simulink, simulate the circuit model described in Fig. 1 and plot the waveforms of $V_i (= [V_{iAB} \ V_{iBC} \ V_{iCA}])$, $I_i (= [i_{iA} \ i_{iB} \ i_{iC}])$, $V_L (= [V_{LAB} \ V_{LBC} \ V_{LCA}])$, and $I_L (= [i_{LA} \ i_{LB} \ i_{LC}])$.

2. Space Vector PWM

2.1 Principle of Pulse Width Modulation (PWM)

Fig. 2 shows circuit model of a single-phase inverter with a center-taped grounded DC bus, and Fig 3 illustrates principle of pulse width modulation.

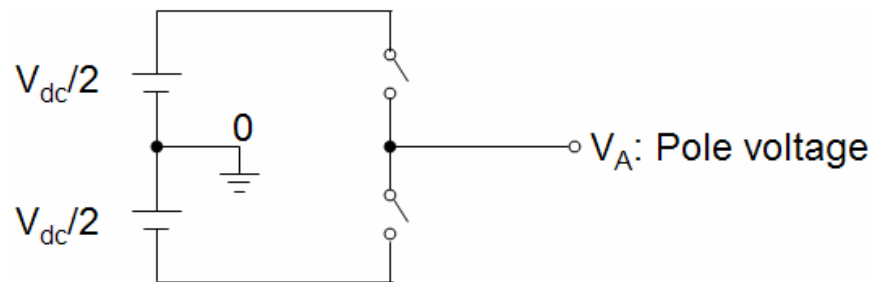


Fig. 2 Circuit model of a single-phase inverter.

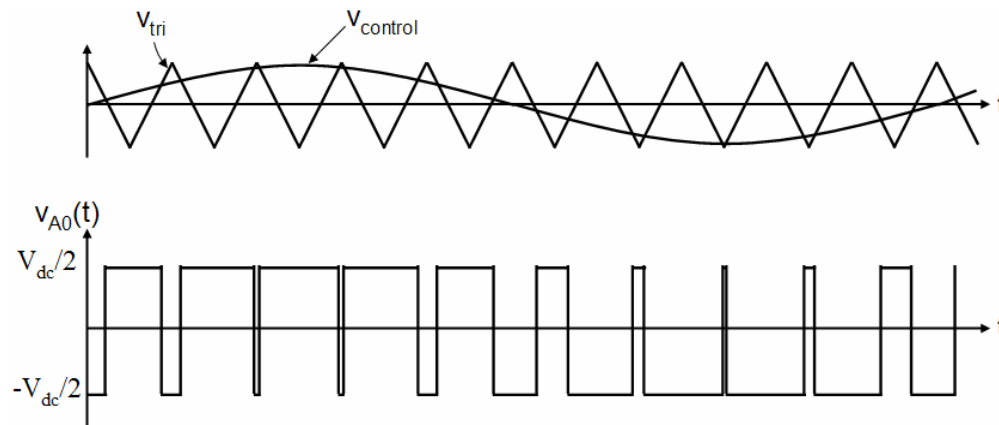


Fig. 3 Pulse width modulation.

As depicted in Fig. 3, the inverter output voltage is determined in the following:

- When $V_{control} > V_{tri}$, $V_{A0} = V_{dc}/2$
- When $V_{control} < V_{tri}$, $V_{A0} = -V_{dc}/2$

Also, the inverter output voltage has the following features:

- PWM frequency is the same as the frequency of V_{tri}
- Amplitude is controlled by the peak value of $V_{control}$
- Fundamental frequency is controlled by the frequency of $V_{control}$

Modulation index (m) is defined as:

$$\therefore m = \frac{v_{control}}{v_{tri}} = \frac{\text{peak of } (V_{A0})_1}{V_{dc}/2},$$

where, $(V_{A0})_1$: fundamental frequency component of V_{A0}

2.2 Principle of Space Vector PWM

The circuit model of a typical three-phase voltage source PWM inverter is shown in Fig. 4. S_1 to S_6 are the six power switches that shape the output, which are controlled by the switching variables a, a', b, b', c and c' . When an upper transistor is switched on, i.e., when a, b or c is 1, the corresponding lower transistor is switched off, i.e., the corresponding a', b' or c' is 0. Therefore, the on and off states of the upper transistors S_1, S_3 and S_5 can be used to determine the output voltage.

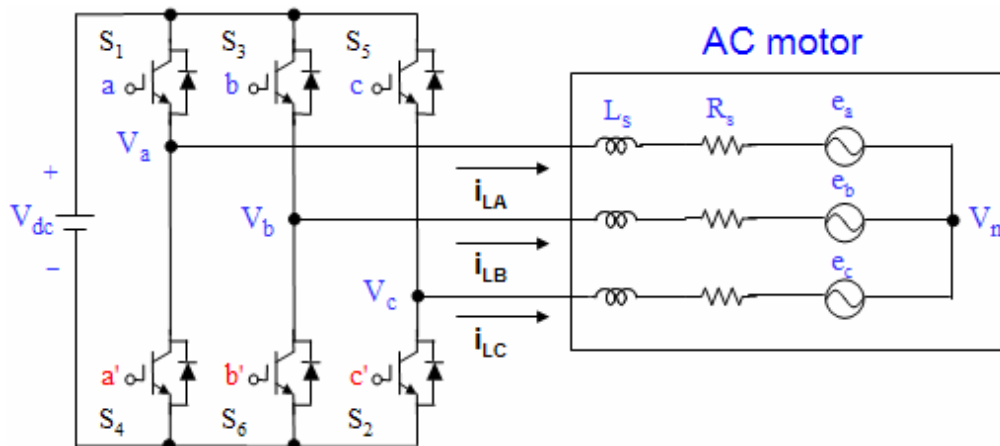


Fig. 4 Three-phase voltage source PWM Inverter.

The relationship between the switching variable vector $[a, b, c]^t$ and the line-to-line voltage vector $[V_{ab} V_{bc} V_{ca}]^t$ is given by (2.1) in the following:

$$\begin{bmatrix} V_{ab} \\ V_{bc} \\ V_{ca} \end{bmatrix} = V_{dc} \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix}. \quad (2.1)$$

Also, the relationship between the switching variable vector $[a, b, c]^t$ and the phase voltage vector $[V_a V_b V_c]^t$ can be expressed below.

$$\begin{bmatrix} V_{an} \\ V_{bn} \\ V_{cn} \end{bmatrix} = \frac{V_{dc}}{3} \begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix}. \quad (2.2)$$

As illustrated in Fig. 4, there are eight possible combinations of on and off patterns for the three upper power switches. The on and off states of the lower power devices are opposite to the upper one and so are easily determined once the states of the upper power transistors are determined. According to equations (2.1) and (2.2), the eight switching vectors, output line to neutral voltage (phase voltage), and output line-to-line voltages in terms of DC-link V_{dc} , are given in Table1 and Fig. 5 shows the eight inverter voltage vectors (V_0 to V_7).

Table 1. Switching vectors, phase voltages and output line to line voltages

Voltage Vectors	Switching Vectors			Line to neutral voltage			Line to line voltage		
	a	b	c	V_{an}	V_{bn}	V_{cn}	V_{ab}	V_{bc}	V_{ca}
V_0	0	0	0	0	0	0	0	0	0
V_1	1	0	0	$2/3$	$-1/3$	$-1/3$	1	0	-1
V_2	1	1	0	$1/3$	$1/3$	$-2/3$	0	1	-1
V_3	0	1	0	$-1/3$	$2/3$	$-1/3$	-1	1	0
V_4	0	1	1	$-2/3$	$1/3$	$1/3$	-1	0	1
V_5	0	0	1	$-1/3$	$-1/3$	$2/3$	0	-1	1
V_6	1	0	1	$1/3$	$-2/3$	$1/3$	1	-1	0
V_7	1	1	1	0	0	0	0	0	0

(Note that the respective voltage should be multiplied by V_{dc})

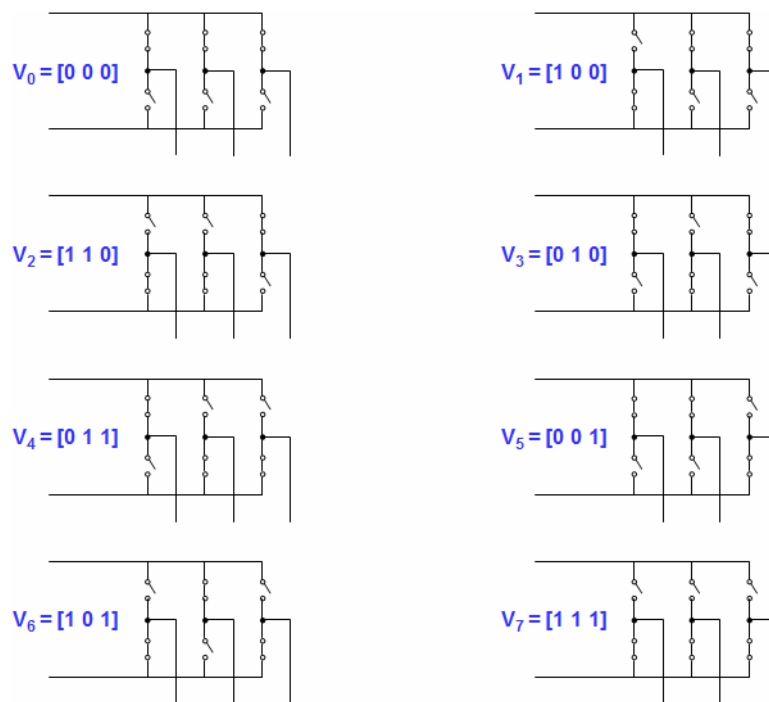


Fig. 5 The eight inverter voltage vectors (V_0 to V_7).

Space Vector PWM (SVPWM) refers to a special switching sequence of the upper three power transistors of a three-phase power inverter. It has been shown to generate less harmonic distortion in the output voltages and or currents applied to the phases of an AC motor and to

provide more efficient use of supply voltage compared with sinusoidal modulation technique as shown in Fig. 6.

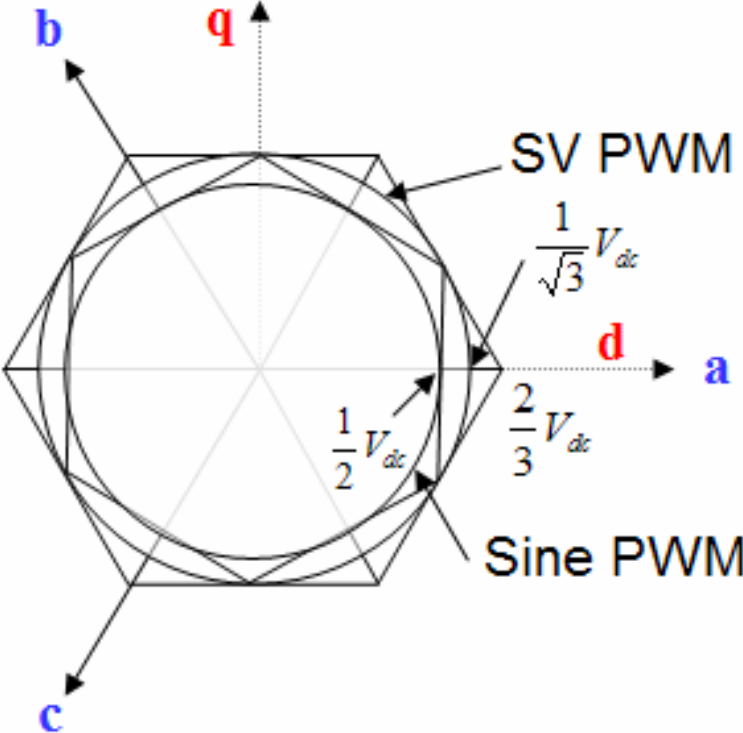


Fig. 6 Locus comparison of maximum linear control voltage in Sine PWM and SVPWM.

To implement the space vector PWM, the voltage equations in the abc reference frame can be transformed into the stationary dq reference frame that consists of the horizontal (d) and vertical (q) axes as depicted in Fig. 7.

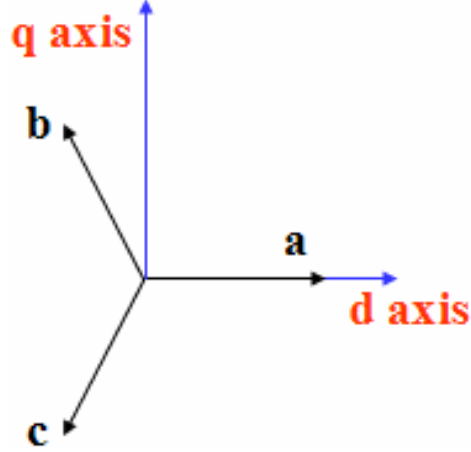


Fig. 7 The relationship of abc reference frame and stationary dq reference frame.

From this figure, the relation between these two reference frames is below

$$\mathbf{f}_{dq0} = \mathbf{K}_s \mathbf{f}_{abc} \quad (2.3)$$

where, $\mathbf{K}_s = \frac{2}{3} \begin{bmatrix} 1 & -1/2 & -1/2 \\ 0 & \sqrt{3}/2 & -\sqrt{3}/2 \\ 1/2 & 1/2 & 1/2 \end{bmatrix}$, $\mathbf{f}_{dq0} = [f_d f_q f_0]^T$, $\mathbf{f}_{abc} = [f_a f_b f_c]^T$, and f denotes either a voltage

or a current variable.

As described in Fig. 7, this transformation is equivalent to an orthogonal projection of $[a, b, c]^t$ onto the two-dimensional perpendicular to the vector $[1, 1, 1]^t$ (the equivalent d-q plane) in a three-dimensional coordinate system. As a result, six non-zero vectors and two zero vectors are possible. Six nonzero vectors ($V_1 - V_6$) shape the axes of a hexagonal as depicted in Fig. 8, and feed electric power to the load. The angle between any adjacent two non-zero vectors is 60 degrees. Meanwhile, two zero vectors (V_0 and V_7) are at the origin and apply zero voltage to the load. The eight vectors are called the basic space vectors and are denoted by $V_0, V_1, V_2, V_3, V_4,$

V_5 , V_6 , and V_7 . The same transformation can be applied to the desired output voltage to get the desired reference voltage vector V_{ref} in the d-q plane.

The objective of space vector PWM technique is to approximate the reference voltage vector V_{ref} using the eight switching patterns. One simple method of approximation is to generate the average output of the inverter in a small period, T to be the same as that of V_{ref} in the same period.

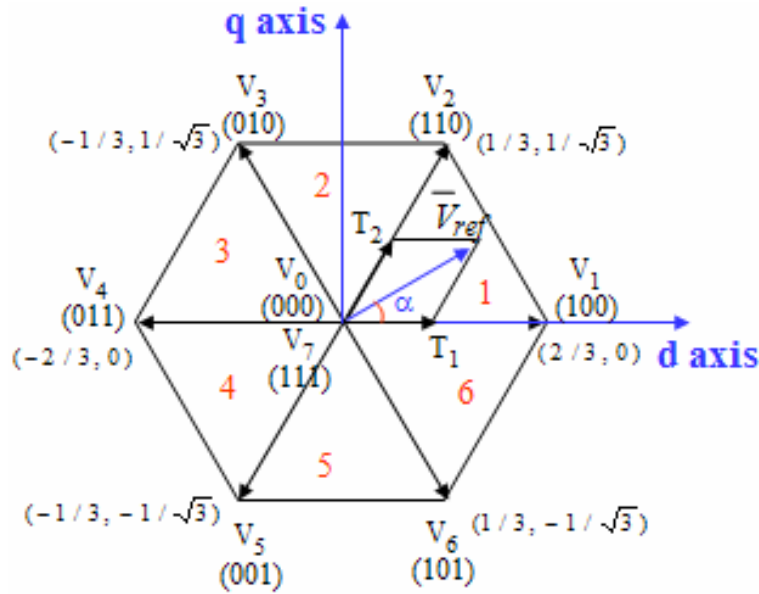


Fig. 8 Basic switching vectors and sectors.

Therefore, space vector PWM can be implemented by the following steps:

- Step 1. Determine V_d , V_q , V_{ref} , and angle (α)
- Step 2. Determine time duration T_1 , T_2 , T_0
- Step 3. Determine the switching time of each transistor (S_1 to S_6)

2.2.1 Step 1: Determine V_d , V_q , V_{ref} , and angle (α)

From Fig. 9, the V_d , V_q , V_{ref} , and angle (α) can be determined as follows:

$$\begin{aligned}
 V_d &= V_{an} - V_{bn} \cdot \cos 60 - V_{cn} \cdot \cos 60 \\
 &= V_{an} - \frac{1}{2} V_{bn} - \frac{1}{2} V_{cn}
 \end{aligned}$$

$$\begin{aligned}
 V_q &= 0 + V_{bn} \cdot \cos 30 - V_{cn} \cdot \cos 30 \\
 &= V_{an} + \frac{\sqrt{3}}{2} V_{bn} - \frac{\sqrt{3}}{2} V_{cn}
 \end{aligned}$$

$$\therefore \begin{bmatrix} V_d \\ V_q \end{bmatrix} = \frac{2}{3} \begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix} \begin{bmatrix} V_{an} \\ V_{bn} \\ V_{cn} \end{bmatrix}$$

$$\therefore |\bar{V}_{ref}| = \sqrt{V_d^2 + V_q^2}$$

$$\therefore \alpha = \tan^{-1} \left(\frac{V_q}{V_d} \right) = \omega t = 2\pi f t, \quad \text{where } f = \text{fundamental frequency}$$

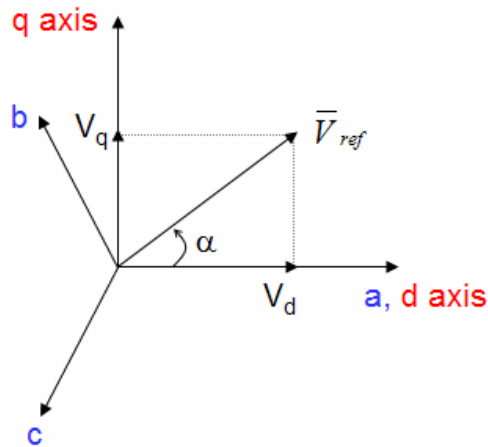


Fig. 9 Voltage Space Vector and its components in (d, q).

2.2.2 Step 2: Determine time duration T_1, T_2, T_0

From Fig. 10, the switching time duration can be calculated as follows:

- Switching time duration at Sector 1

$$\int_0^{T_z} \bar{V}_{ref} dt = \int_0^{T_1} \bar{V}_1 dt + \int_{T_1}^{T_1+T_2} \bar{V}_2 dt + \int_{T_1+T_2}^{T_z} \bar{V}_0 dt$$

$$\therefore T_z \cdot \bar{V}_{ref} = (T_1 \cdot \bar{V}_1 + T_2 \cdot \bar{V}_2)$$

$$\Rightarrow T_z \cdot |\bar{V}_{ref}| \cdot \begin{bmatrix} \cos(\alpha) \\ \sin(\alpha) \end{bmatrix} = T_1 \cdot \frac{2}{3} \cdot V_{dc} \cdot \begin{bmatrix} 1 \\ 0 \end{bmatrix} + T_2 \cdot \frac{2}{3} \cdot V_{dc} \cdot \begin{bmatrix} \cos(\pi/3) \\ \sin(\pi/3) \end{bmatrix}$$

(where, $0 \leq \alpha \leq 60^\circ$)

$$\therefore T_1 = T_z \cdot a \cdot \frac{\sin(\pi/3 - \alpha)}{\sin(\pi/3)}$$

$$\therefore T_2 = T_z \cdot a \cdot \frac{\sin(\alpha)}{\sin(\pi/3)}$$

$$\therefore T_0 = T_z - (T_1 + T_2), \quad \left(\text{where, } T_z = \frac{1}{f_z} \text{ and } a = \frac{|\bar{V}_{ref}|}{\frac{2}{3} V_{dc}} \right)$$

- Switching time duration at any Sector

$$\therefore T_1 = \frac{\sqrt{3} \cdot T_z \cdot |\bar{V}_{ref}|}{V_{dc}} \left(\sin \left(\frac{\pi}{3} - \alpha + \frac{n-1}{3} \pi \right) \right)$$

$$= \frac{\sqrt{3} \cdot T_z \cdot |\bar{V}_{ref}|}{V_{dc}} \left(\sin \frac{n}{3} \pi - \alpha \right)$$

$$= \frac{\sqrt{3} \cdot T_z \cdot |\bar{V}_{ref}|}{V_{dc}} \left(\sin \frac{n}{3} \pi \cos \alpha - \cos \frac{n}{3} \pi \sin \alpha \right)$$

$$\therefore T_2 = \frac{\sqrt{3} \cdot T_z \cdot |\bar{V}_{ref}|}{V_{dc}} \left(\sin \left(\alpha - \frac{n-1}{3} \pi \right) \right)$$

$$= \frac{\sqrt{3} \cdot T_z \cdot |\bar{V}_{ref}|}{V_{dc}} \left(-\cos \alpha \cdot \sin \frac{n-1}{3} \pi + \sin \alpha \cdot \cos \frac{n-1}{3} \pi \right)$$

$$\therefore T_0 = T_z - T_1 - T_2, \quad \left(\text{where, } n = 1 \text{ through } 6 \text{ (that is, Sector 1 to 6)} \right)$$

$0 \leq \alpha \leq 60^\circ$

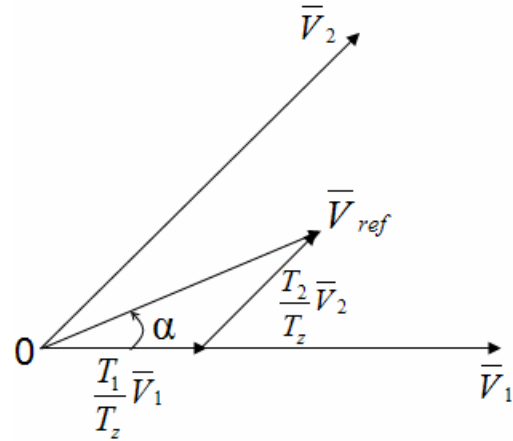
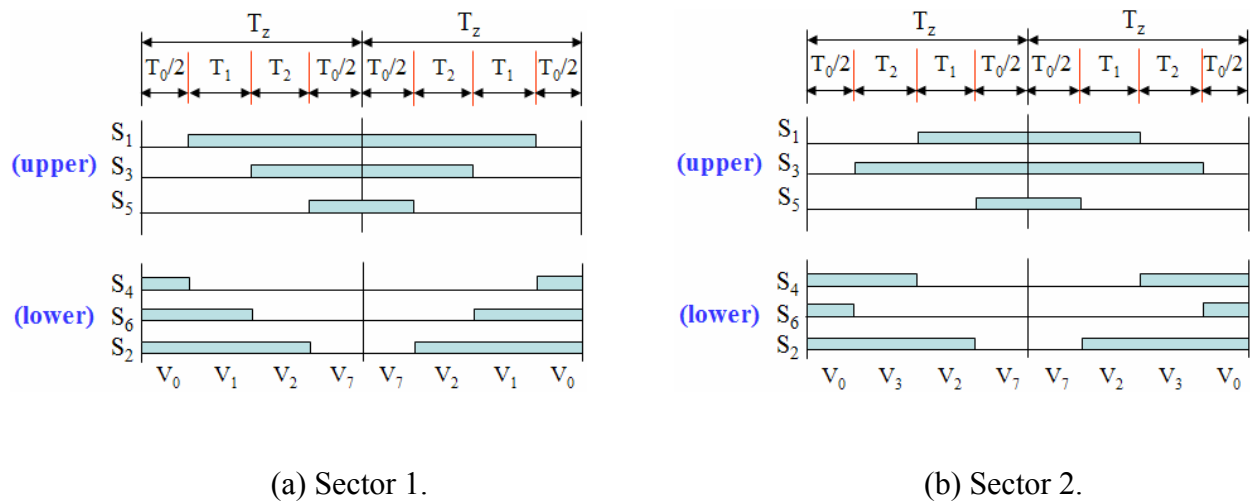
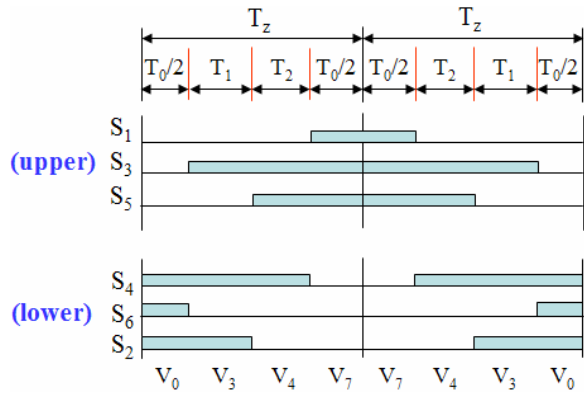


Fig. 10 Reference vector as a combination of adjacent vectors at sector 1.

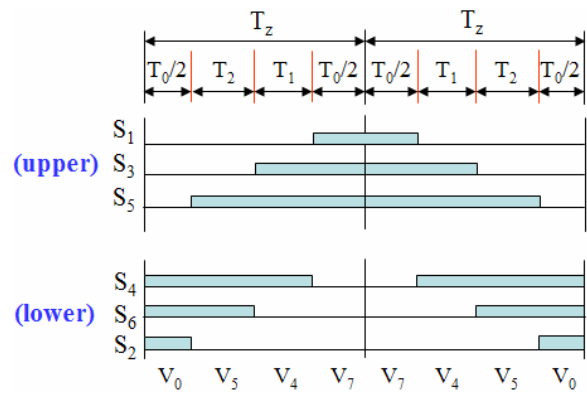
2.2.3 Step 3: Determine the switching time of each transistor (S₁ to S₆)

Fig. 11 shows space vector PWM switching patterns at each sector.

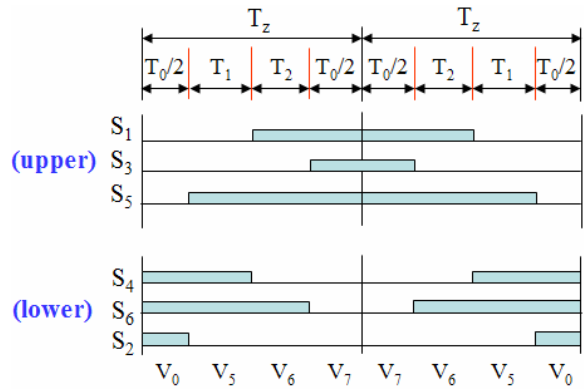




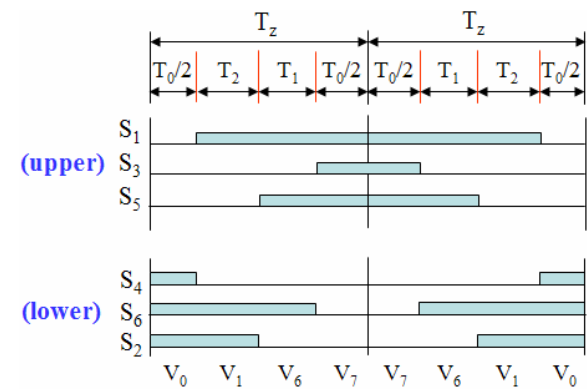
(c) Sector 3.



(d) Sector 4.



(e) Sector 5.



(f) Sector 6.

Fig. 11 Space Vector PWM switching patterns at each sector.

Based on Fig. 11, the switching time at each sector is summarized in Table 2, and it will be built in Simulink model to implement SVPWM.

Table 2. Switching Time Calculation at Each Sector

Sector	Upper Switches (S_1, S_3, S_5)	Lower Switches (S_4, S_6, S_2)
1	$S_1 = T_1 + T_2 + T_0 / 2$ $S_3 = T_2 + T_0 / 2$ $S_5 = T_0 / 2$	$S_4 = T_0 / 2$ $S_6 = T_1 + T_0 / 2$ $S_2 = T_1 + T_2 + T_0 / 2$
2	$S_1 = T_1 + T_0 / 2$ $S_3 = T_1 + T_2 + T_0 / 2$ $S_5 = T_0 / 2$	$S_4 = T_2 + T_0 / 2$ $S_6 = T_0 / 2$ $S_2 = T_1 + T_2 + T_0 / 2$
3	$S_1 = T_0 / 2$ $S_3 = T_1 + T_2 + T_0 / 2$ $S_5 = T_2 + T_0 / 2$	$S_4 = T_1 + T_2 + T_0 / 2$ $S_6 = T_0 / 2$ $S_2 = T_1 + T_0 / 2$
4	$S_1 = T_0 / 2$ $S_3 = T_1 + T_0 / 2$ $S_5 = T_1 + T_2 + T_0 / 2$	$S_4 = T_1 + T_2 + T_0 / 2$ $S_6 = T_2 + T_0 / 2$ $S_2 = T_0 / 2$
5	$S_1 = T_2 + T_0 / 2$ $S_3 = T_0 / 2$ $S_5 = T_1 + T_2 + T_0 / 2$	$S_4 = T_1 + T_0 / 2$ $S_6 = T_1 + T_2 + T_0 / 2$ $S_2 = T_0 / 2$
6	$S_1 = T_1 + T_2 + T_0 / 2$ $S_3 = T_0 / 2$ $S_5 = T_1 + T_0 / 2$	$S_4 = T_0 / 2$ $S_6 = T_1 + T_2 + T_0 / 2$ $S_2 = T_2 + T_0 / 2$

3. State-Space Model

Fig. 12 shows L-C output filter to obtain current and voltage equations.

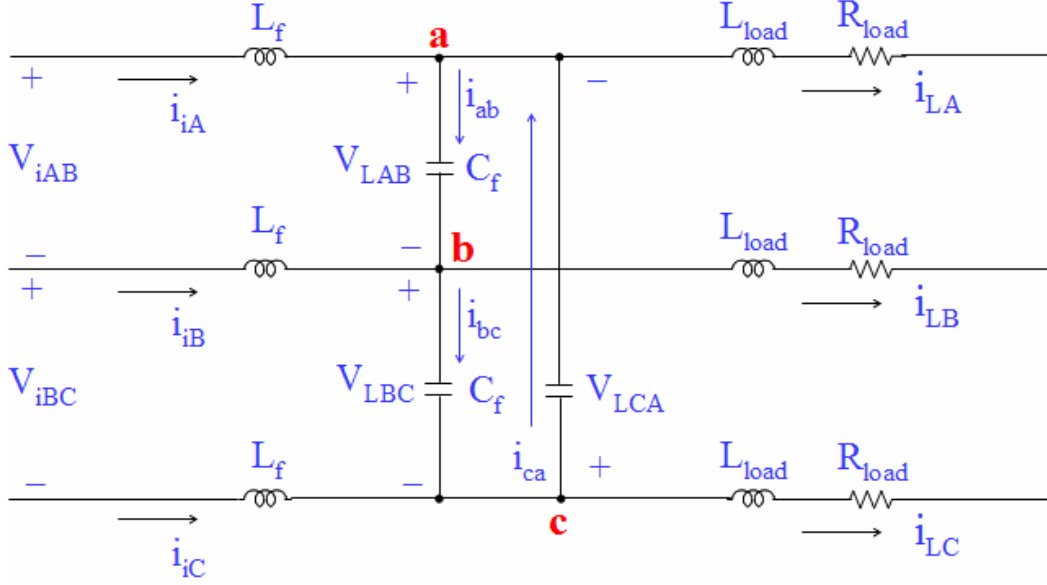


Fig. 12 L-C output filter for current/voltage equations.

By applying Kirchoff's current law to nodes a, b, and c, respectively, the following current equations are derived:

① node "a":

$$i_{iA} + i_{ca} = i_{ab} + i_{LA} \Rightarrow i_{iA} + C_f \frac{dV_{LCA}}{dt} = C_f \frac{dV_{LAB}}{dt} + i_{LA}. \quad (3.1)$$

② node "b":

$$i_{iB} + i_{ab} = i_{bc} + i_{LB} \Rightarrow i_{iB} + C_f \frac{dV_{LAB}}{dt} = C_f \frac{dV_{LBC}}{dt} + i_{LB}. \quad (3.2)$$

③ node "c":

$$i_{iC} + i_{bc} = i_{ca} + i_{LC} \Rightarrow i_{iC} + C_f \frac{dV_{LBC}}{dt} = C_f \frac{dV_{LCA}}{dt} + i_{LC}. \quad (3.2)$$

where, $i_{ab} = C_f \frac{dV_{LAB}}{dt}$, $i_{bc} = C_f \frac{dV_{LBC}}{dt}$, $i_{ca} = C_f \frac{dV_{LCA}}{dt}$.

Also, (3.1) to (3.3) can be rewritten as the following equations, respectively:

① subtracting (3.2) from (3.1):

$$\begin{aligned} i_{iA} - i_{iB} + C_f \left(\frac{dV_{LCA}}{dt} - \frac{dV_{LAB}}{dt} \right) &= C_f \left(\frac{dV_{LAB}}{dt} - \frac{dV_{LBC}}{dt} \right) + i_{LA} - i_{LB} \\ \Rightarrow C_f \left(\frac{dV_{LCA}}{dt} + \frac{dV_{LBC}}{dt} - 2 \cdot \frac{dV_{LAB}}{dt} \right) &= -i_{iA} + i_{iB} + i_{LA} - i_{LB} \end{aligned} \quad (3.4)$$

② subtracting (3.3) from (3.2):

$$\begin{aligned} i_{iB} - i_{iC} + C_f \left(\frac{dV_{LAB}}{dt} - \frac{dV_{LBC}}{dt} \right) &= C_f \left(\frac{dV_{LBC}}{dt} - \frac{dV_{LCA}}{dt} \right) + i_{LB} - i_{LC} \\ \Rightarrow C_f \left(\frac{dV_{LAB}}{dt} + \frac{dV_{LCA}}{dt} - 2 \cdot \frac{dV_{LBC}}{dt} \right) &= -i_{iB} + i_{iC} + i_{LB} - i_{LC} \end{aligned} \quad (3.5)$$

③ subtracting (3.1) from (3.3):

$$\begin{aligned} i_{iC} - i_{iA} + C_f \left(\frac{dV_{LBC}}{dt} - \frac{dV_{LCA}}{dt} \right) &= C_f \left(\frac{dV_{LCA}}{dt} - \frac{dV_{LAB}}{dt} \right) + i_{LC} - i_{LA} \\ \Rightarrow C_f \left(\frac{dV_{LAB}}{dt} + \frac{dV_{LBC}}{dt} - 2 \cdot \frac{dV_{LCA}}{dt} \right) &= -i_{iC} + i_{iA} + i_{LC} - i_{LA} \end{aligned} \quad (3.6)$$

To simplify (3.4) to (3.6), we use the following relationship that an algebraic sum of line to line load voltages is equal to zero:

$$V_{LAB} + V_{LBC} + V_{LCA} = 0. \quad (3.7)$$

Based on (3.7), the (3.4) to (3.6) can be modified to a first-order differential equation, respectively:

$$\begin{cases} \frac{dV_{LAB}}{dt} = \frac{1}{3C_f} i_{iAB} - \frac{1}{3C_f} (i_{LAB}) \\ \frac{dV_{LBC}}{dt} = \frac{1}{3C_f} i_{iBC} - \frac{1}{3C_f} (i_{LBC}), \\ \frac{dV_{LCA}}{dt} = \frac{1}{3C_f} i_{iCA} - \frac{1}{3C_f} (i_{LCA}) \end{cases} \quad (3.8)$$

where, $i_{iAB} = i_{iA} - i_{iB}$, $i_{iBC} = i_{iB} - i_{iC}$, $i_{iCA} = i_{iC} - i_{iA}$ and $i_{LAB} = i_{LA} - i_{LB}$, $i_{LBC} = i_{LB} - i_{LC}$,
 $i_{LCA} = i_{LC} - i_{LA}$.

By applying Kirchoff's voltage law on the side of inverter output, the following voltage equations can be derived:

$$\begin{cases} \frac{di_{iAB}}{dt} = -\frac{1}{L_f} V_{LAB} + \frac{1}{L_f} V_{iAB} \\ \frac{di_{iBC}}{dt} = -\frac{1}{L_f} V_{LBC} + \frac{1}{L_f} V_{iBC} \cdot \\ \frac{di_{iCA}}{dt} = -\frac{1}{L_f} V_{LCA} + \frac{1}{L_f} V_{iCA} \end{cases} \quad (3.9)$$

By applying Kirchoff's voltage law on the load side, the following voltage equations can be derived:

$$\begin{cases} V_{LAB} = L_{load} \frac{di_{LA}}{dt} + R_{load} i_{LA} - L_{load} \frac{di_{LB}}{dt} - R_{load} i_{LB} \\ V_{LBC} = L_{load} \frac{di_{LB}}{dt} + R_{load} i_{LB} - L_{load} \frac{di_{LC}}{dt} - R_{load} i_{LC} \cdot \\ V_{LCA} = L_{load} \frac{di_{LC}}{dt} + R_{load} i_{LC} - L_{load} \frac{di_{LA}}{dt} - R_{load} i_{LA} \end{cases} \quad (3.10)$$

Equation (3.10) can be rewritten as:

$$\begin{cases} \frac{di_{LAB}}{dt} = -\frac{R_{load}}{L_{load}}i_{LAB} + \frac{1}{L_{load}}V_{LAB} \\ \frac{di_{LBC}}{dt} = -\frac{R_{load}}{L_{load}}i_{LBC} + \frac{1}{L_{load}}V_{LBC} \\ \frac{di_{LCA}}{dt} = -\frac{R_{load}}{L_{load}}i_{LCA} + \frac{1}{L_{load}}V_{LCA} \end{cases} \quad (3.11)$$

Therefore, we can rewrite (3.8), (3.9) and (3.11) into a matrix form, respectively:

$$\begin{aligned} \frac{d\mathbf{V}_L}{dt} &= \frac{1}{3C_f}\mathbf{I}_i - \frac{1}{3C_f}\mathbf{I}_L \\ \frac{d\mathbf{I}_i}{dt} &= -\frac{1}{L_f}\mathbf{V}_L + \frac{1}{L_f}\mathbf{V}_i, \\ \frac{d\mathbf{I}_L}{dt} &= \frac{1}{L_{load}}\mathbf{V}_L - \frac{R_{load}}{L_{load}}\mathbf{I}_L \end{aligned} \quad (3.12)$$

where, $\mathbf{V}_L = [V_{LAB} \ V_{LBC} \ V_{LCA}]^T$, $\mathbf{I}_i = [i_{iAB} \ i_{iBC} \ i_{iCA}]^T = [i_{iA}-i_{iB} \ i_{iB}-i_{iC} \ i_{iC}-i_{iA}]^T$, $\mathbf{V}_i = [V_{iAB} \ V_{iBC} \ V_{iCA}]^T$, $\mathbf{I}_L = [i_{LAB} \ i_{LBC} \ i_{LCA}]^T = [i_{LA}-i_{LB} \ i_{LB}-i_{LC} \ i_{LC}-i_{LA}]^T$.

Finally, the given plant model (3.12) can be expressed as the following continuous-time state space equation

$$\dot{\mathbf{X}}(t) = \mathbf{A}\mathbf{X}(t) + \mathbf{B}\mathbf{u}(t), \quad (3.13)$$

$$\text{where, } \mathbf{X} = \begin{bmatrix} \mathbf{V}_L \\ \mathbf{I}_i \\ \mathbf{I}_L \end{bmatrix}_{9 \times 1}, \mathbf{A} = \begin{bmatrix} \mathbf{0}_{3 \times 3} & \frac{1}{3C_f}I_{3 \times 3} & -\frac{1}{3C_f}I_{3 \times 3} \\ -\frac{1}{L_f}I_{3 \times 3} & \mathbf{0}_{3 \times 3} & \mathbf{0}_{3 \times 3} \\ \frac{1}{L_{load}}I_{3 \times 3} & \mathbf{0}_{3 \times 3} & -\frac{R_{load}}{L_{load}}I_{3 \times 3} \end{bmatrix}_{9 \times 9}, \mathbf{B} = \begin{bmatrix} \mathbf{0}_{3 \times 3} \\ \frac{1}{L_f}I_{3 \times 3} \\ \mathbf{0}_{3 \times 3} \end{bmatrix}_{9 \times 3}, \mathbf{u} = [\mathbf{V}_i]_{3 \times 1}.$$

Note that load line to line voltage \mathbf{V}_L , inverter output current \mathbf{I}_i , and the load current \mathbf{I}_L are the state variables of the system, and the inverter output line-to-line voltage \mathbf{V}_i is the control input (\mathbf{u}).

4. Simulation Steps

1). Initialize system parameters using Matlab

2). Build Simulink Model

- Determine sector
- Determine time duration T_1, T_2, T_0
- Determine the switching time ($T_a, T_b,$ and T_c) of each transistor (S_1 to S_6)
- Generate the inverter output voltages ($V_{iAB}, V_{iBC}, V_{iCA},$) for control input (u)
- Send data to Workspace

3). Plot simulation results using Matlab

5. Simulation results

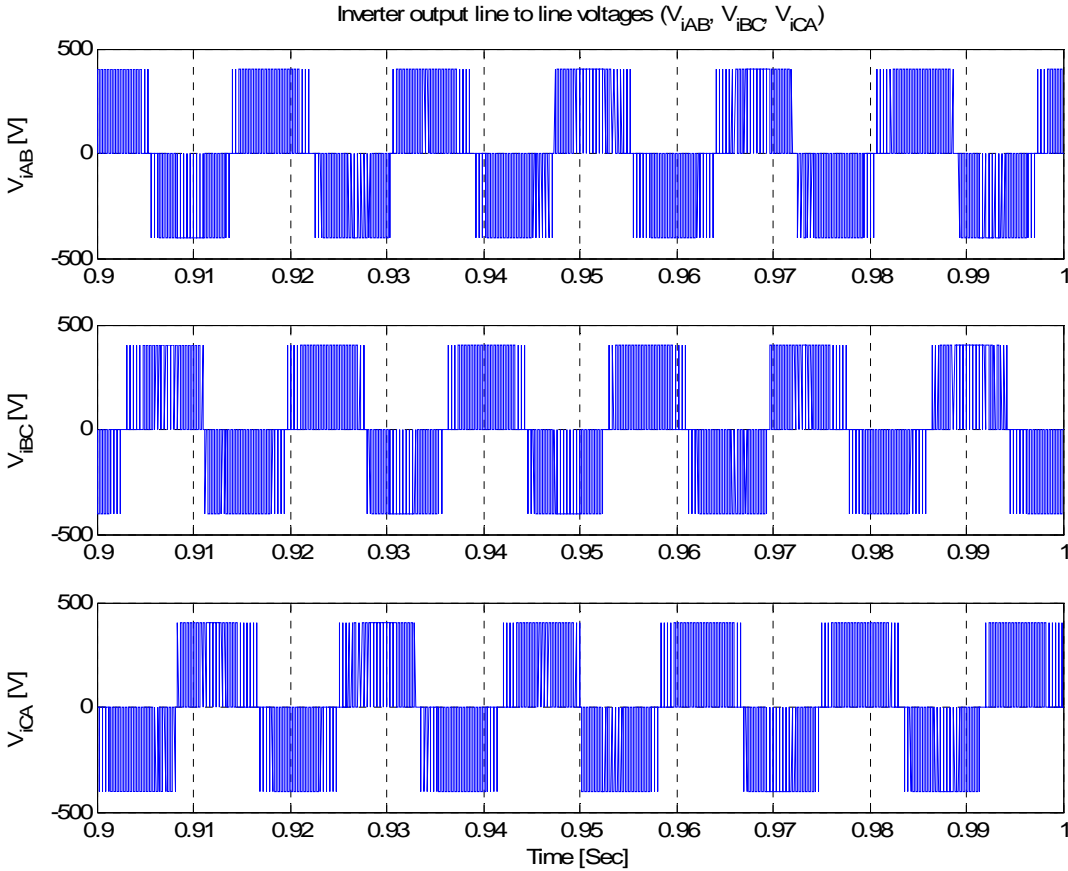


Fig. 13 Simulation results of inverter output line to line voltages (V_{iAB} , V_{iBC} , V_{iCA})

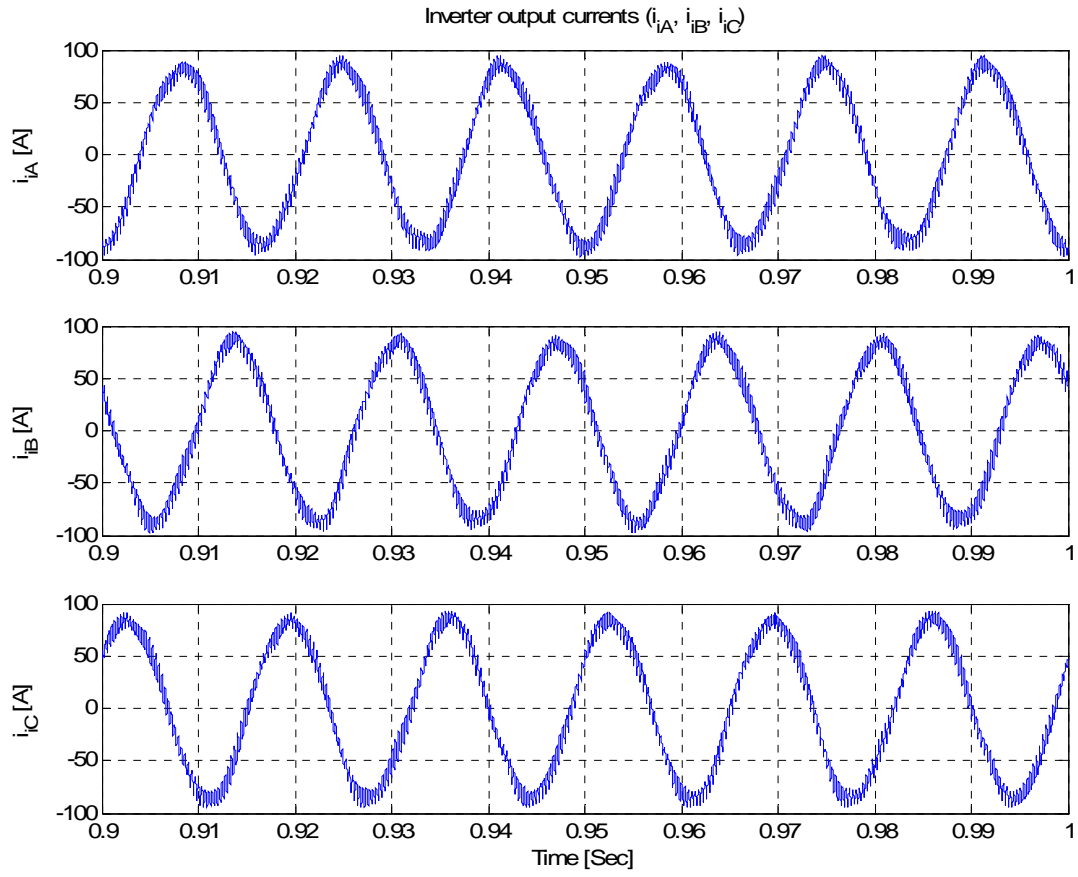


Fig. 14 Simulation results of inverter output currents (i_A , i_B , i_C)

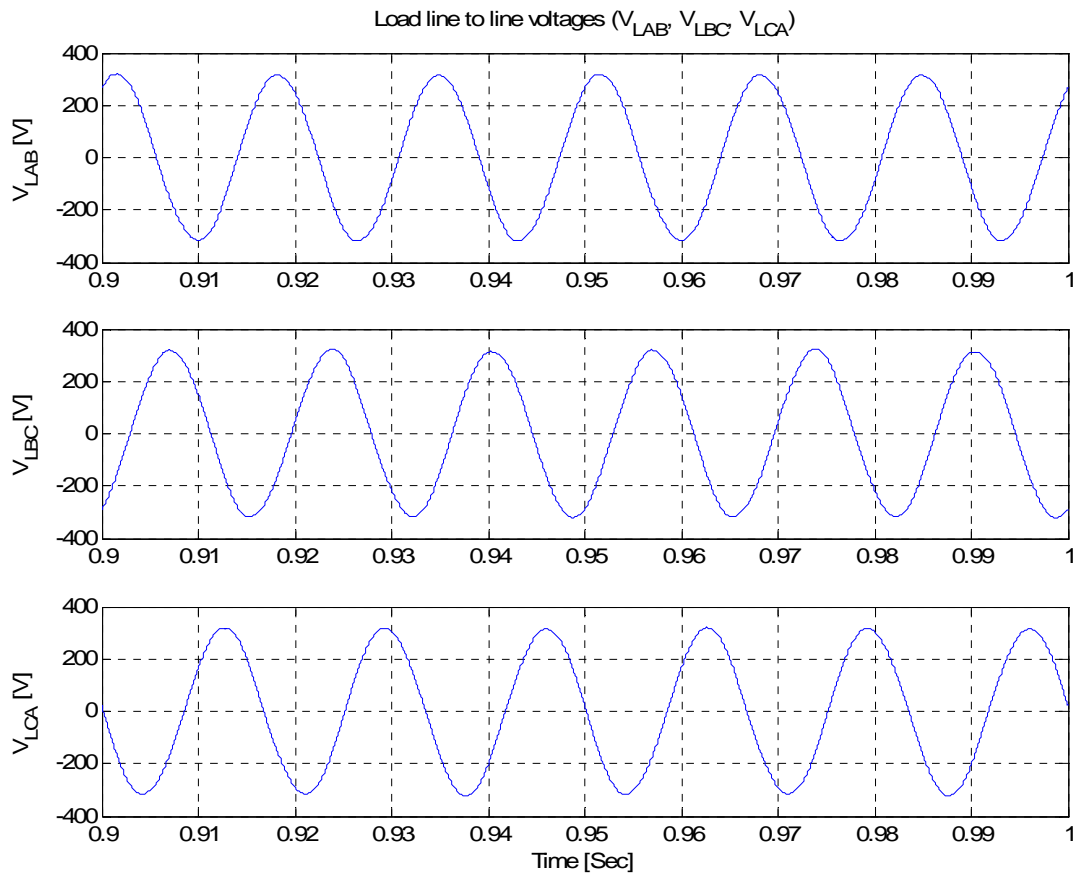


Fig. 15 Simulation results of load line to line voltages (V_{LAB} , V_{LBC} , V_{LCA})

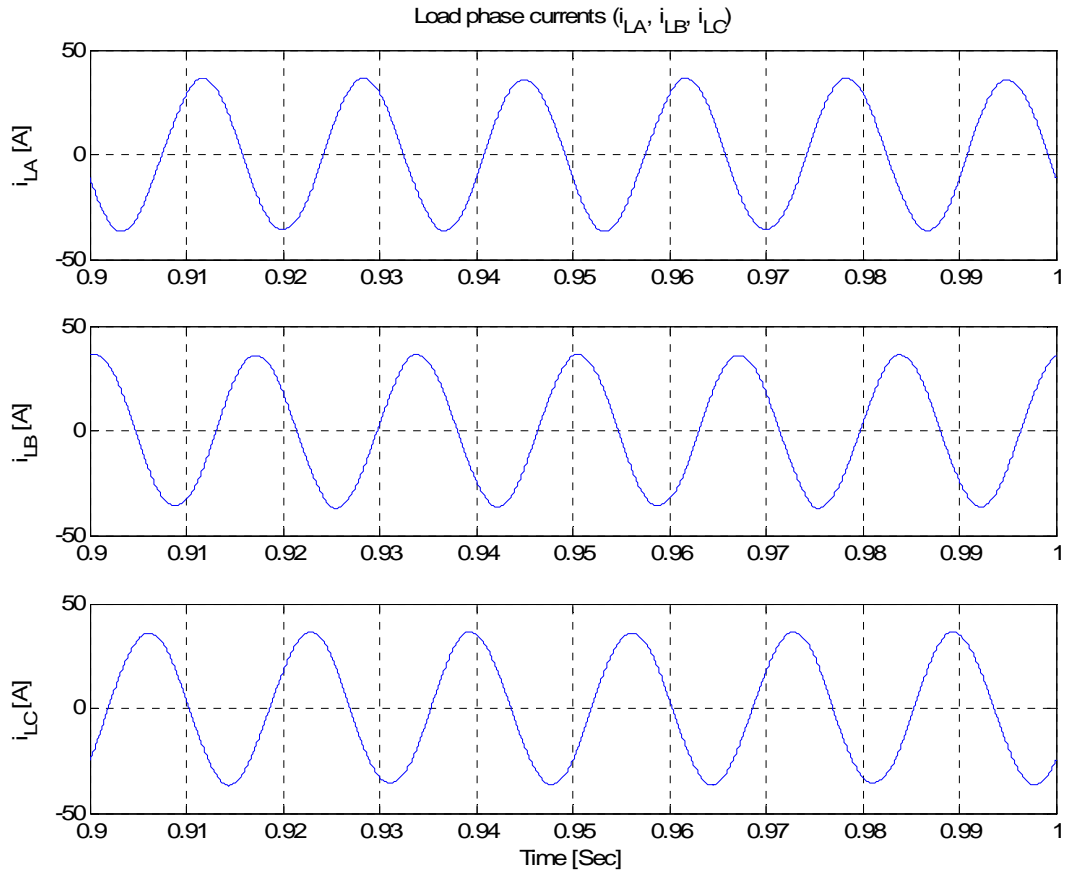


Fig. 16 Simulation results of load phase currents (i_{LA} , i_{LB} , i_{LC})

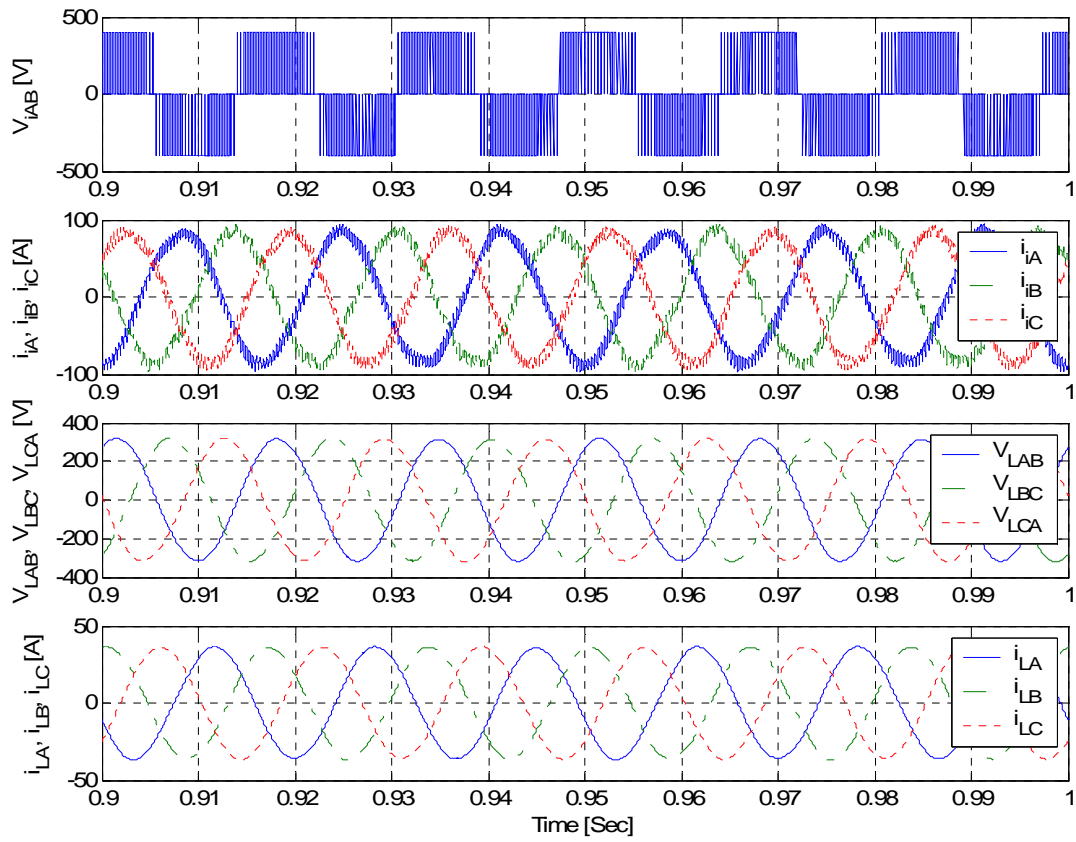


Fig. 17 Simulation waveforms.

- (a) Inverter output line to line voltage (V_{iAB})
- (b) Inverter output current (i_{iA})
- (c) Load line to line voltage (V_{LAB})
- (d) Load phase current (i_{LA})

Appendix

Matlab/Simulink Codes

A.1 Matlab Code for System Parameters

% Written by Jin Woo Jung

% Date: 02/20/05

% ECE743, Simulation Project #2 (Space Vector PWM Inverter)

% Matlab program for Parameter Initialization

clear all % clear workspace

% Input data

Vdc= 400; % DC-link voltage

Lf= 800e-6;% Inductance for output filter

Cf= 400e-6; % Capacitance for output filter

Lload = 2e-3; %Load inductance

Rload= 5; % Load resistance

f= 60; % Fundamental frequency

fz = 3e3; % Switching frequency

a= 0.6;% Modulation index

w= 2*pi*60; %angular frequency

Tz= 1/fz; % Sampling time

V_ref= (2/3)*a*Vdc; % Reference voltage

% Coefficients for State-Space Model

A=[zeros(3,3) eye(3)/(3*Cf) -eye(3)/(3*Cf)

-eye(3)/Lf zeros(3,3) zeros(3,3)

eye(3,3)/Lload zeros(3,3) -eye(3)*Rload/Lload]; % system matrix

B=[zeros(3,3)

eye(3)/Lf

zeros(3,3)]; % coefficient for the control variable u

```
C=[eye(9)]; % coefficient for the output y
```

```
D=[zeros(9,3)]; % coefficient for the output y
```

```
Ks = 1/3*[-1 0 1; 1 -1 0; 0 1 -1]; % Conversion matrix to transform [iiAB iiBC iiCA] to [iiA iiB iiC]
```

A.2 Matlab Code for Plotting the Simulation Results

% Written by Jin Woo Jung

% Date: 02/20/05

% ECE743, Simulation Project #2 (Space Vector PWM)

% Matlab program for plotting Simulation Results

% using Simulink

ViAB = Vi(:,1);

ViBC = Vi(:,2);

ViCA = Vi(:,3);

VLAB= VL(:,1);

VLBC= VL(:,2);

VLCA= VL(:,3);

iiA= IiABC(:,1);

iiB= IiABC(:,2);

iiC= IiABC(:,3);

iLA= ILABC(:,1);

iLB= ILABC(:,2);

iLC= ILABC(:,3);

figure(1)

subplot(3,1,1)

plot(t,ViAB)

axis([0.9 1 -500 500])

ylabel('V_i_A_B [V]')

title('Inverter output line to line voltages (V_i_A_B, V_i_B_C, V_i_C_A)')

```
grid
```

```
subplot(3,1,2)  
plot(t,ViBC)  
axis([0.9 1 -500 500])  
ylabel('V_i_B_C [V]')  
grid
```

```
subplot(3,1,3)  
plot(t,ViCA)  
axis([0.9 1 -500 500])  
ylabel('V_i_C_A [V]')  
xlabel('Time [Sec]')  
grid
```

```
figure(2)  
subplot(3,1,1)  
plot(t,iiA)  
axis([0.9 1 -100 100])  
ylabel('i_i_A [A]')  
title('Inverter output currents (i_i_A, i_i_B, i_i_C)')  
grid
```

```
subplot(3,1,2)  
plot(t,iiB)  
axis([0.9 1 -100 100])  
ylabel('i_i_B [A]')  
grid
```

```
subplot(3,1,3)
```

```
plot(t,iiC)
axis([0.9 1 -100 100])
ylabel('ii_C [A]')
xlabel('Time [Sec]')
grid
```

```
figure(3)
subplot(3,1,1)
plot(t,VLAB)
axis([0.9 1 -400 400])
ylabel('VL_A_B [V]')
title('Load line to line voltages (VL_A_B, VL_B_C, VL_C_A)')
grid
```

```
subplot(3,1,2)
plot(t,VLBC)
axis([0.9 1 -400 400])
ylabel('VL_B_C [V]')
grid
```

```
subplot(3,1,3)
plot(t,VLCA)
axis([0.9 1 -400 400])
ylabel('VL_C_A [V]')
xlabel('Time [Sec]')
grid
```

```
figure(4)
subplot(3,1,1)
plot(t,iLA)
axis([0.9 1 -50 50])
```

```
ylabel('i_L_A [A]')
title('Load phase currents (i_L_A, i_L_B, i_L_C)')
grid
```

```
subplot(3,1,2)
plot(t,iLB)
axis([0.9 1 -50 50])
ylabel('i_L_B [A]')
grid
```

```
subplot(3,1,3)
plot(t,iLC)
axis([0.9 1 -50 50])
ylabel('i_L_C [A]')
xlabel('Time [Sec]')
grid
```

```
figure(5)
subplot(4,1,1)
plot(t,ViAB)
axis([0.9 1 -500 500])
ylabel('V_i_A_B [V]')
grid
```

```
subplot(4,1,2)
plot(t,iiA,'-', t,iiB,'-.',t,iiC,'!')
axis([0.9 1 -100 100])
ylabel('i_i_A, i_i_B, i_i_C [A]')
legend('i_i_A', 'i_i_B', 'i_i_C')
grid
```

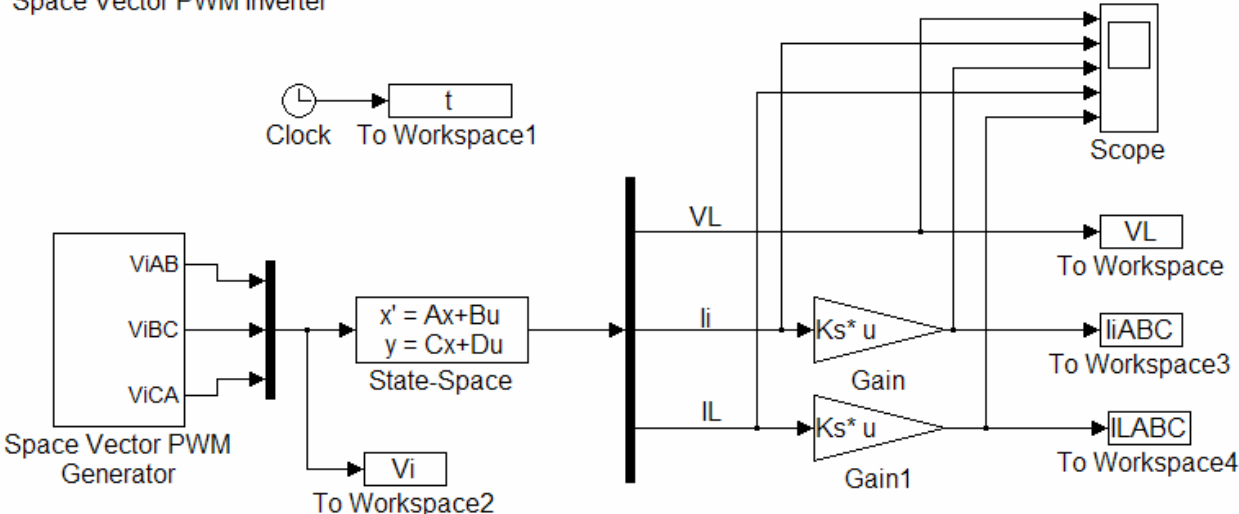
```
subplot(4,1,3)
plot(t,VLAB,'-', t,VLBC,'-',t,VLCA,':')
axis([0.9 1 -400 400])
ylabel('V_L_A_B, V_L_B_C, V_L_C_A [V]')
legend('V_L_A_B', 'V_L_B_C', 'V_L_C_A')
grid
```

```
subplot(4,1,4)
plot(t,iLA,'-', t,iLB,'-',t,iLC,':')
axis([0.9 1 -50 50])
ylabel('i_L_A, i_L_B, i_L_C [A]')
legend('i_L_A', 'i_L_B', 'i_L_C')
xlabel('Time [Sec]')
grid
```

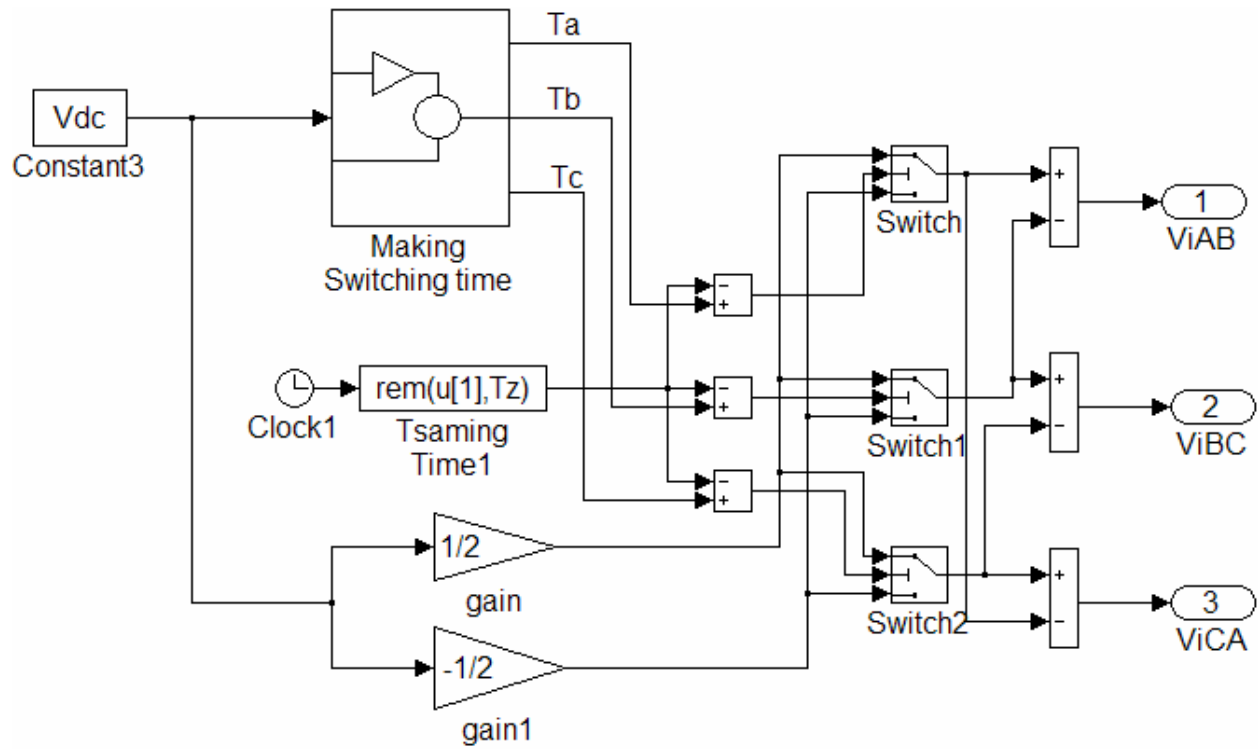

A.3 Simulink Code

Simulink Model for Overall System

Date: 02/20/05
 Written by Jin Woo Jung
 ECE743: Simulation Project #2
 Space Vector PWM Inverter

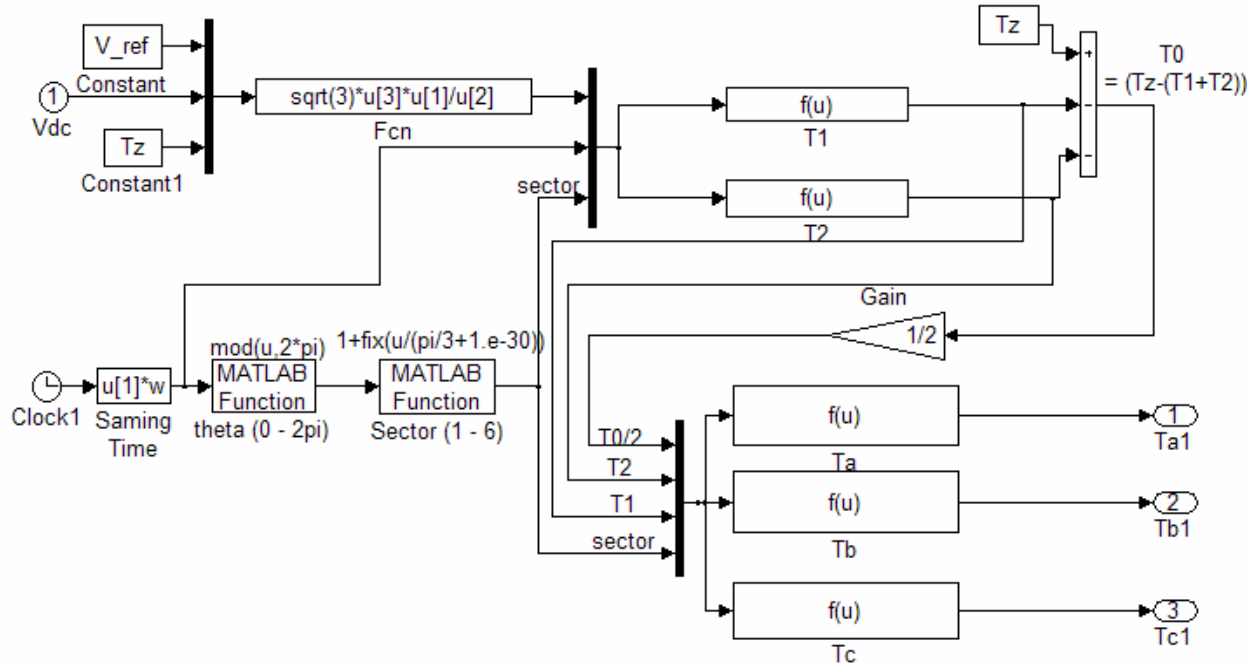


Subsystem Simulink Model for “Space Vector PWM Generator”



Subsystem Simulink Model for “Making Switching Time”

Determine PWM switching time



$$1). \mathbf{T1} = u[1] * (\sin(u[3] * \pi/3) * \cos(u[2]) - \cos(u[3] * \pi/3) * \sin(u[2]))$$

$$2). \mathbf{T2} = u[1] * (\cos((u[3]-1) * (\pi/3)) * \sin(u[2]) - \sin((u[3]-1) * (\pi/3)) * \cos(u[2]))$$

$$3). \mathbf{T_a} = (u[4]==1) * (u[1]+u[2]+u[3]) + (u[4]==2) * (u[1]+u[2]+u[3]) + (u[4]==3) * (u[1]+u[3]) + (u[4]==4) * (u[1]) + (u[4]==5) * (u[1]) + (u[4]==6) * (u[1]+u[2])$$

$$4). \mathbf{T_b} = (u[4]==1) * (u[1]) + (u[4]==2) * (u[1]+u[2]) + (u[4]==3) * (u[1]+u[2]+u[3]) + (u[4]==4) * (u[1]+u[2]+u[3]) + (u[4]==5) * (u[1]+u[3]) + (u[4]==6) * (u[1])$$

$$5). \mathbf{T_c} = (u[4]==1) * (u[1]+u[3]) + (u[4]==2) * (u[1]) + (u[4]==3) * (u[1]) + (u[4]==4) * (u[1]+u[2]) + (u[4]==5) * (u[1]+u[2]+u[3]) + (u[4]==6) * (u[1]+u[2]+u[3])$$